ISSN 0021-3640, JETP Letters, 2008, Vol. 87, No. 4, pp. 213–214. © Pleiades Publishing, Ltd., 2008.
Original Russian Text © V.I. Marchenko, 2008, published in Pis'ma v Zhurnal Éksperimental'noĭ i Teoreticheskoĭ Fiziki, 2008, Vol. 87, No. 4, pp. 245–246.

On the Phases of Compressed Matter

V. I. Marchenko

Kapitza Institute for Physical Problems, Russian Academy of Sciences, ul. Kosygina 2, Moscow, 119334 Russia

Received December 10, 2007; in final form, January 11, 2008

It is shown that matter during the process of compression under the conditions of complete thermodynamic equilibrium at low temperatures should sequentially pass through 14 monoisotopic phase states. The nuclear subsystem is a Wigner crystal in almost all phases and is a quantum liquid at the maximum pressure.

PACS numbers: 64.90.+b

DOI: 10.1134/S0021364008040073

Matter observed under normal (terrestrial) conditions is usually in a metastable state with respect to nuclear reactions. Matter subjected to strong compression, when internuclear distances become noticeably smaller than the atomic size, but are larger than the nuclear radius electrons, becomes uncoupled with nuclei and forms an almost ideal Fermi gas [1]. The thermal motion of the nuclei leads to their collisions and the equilibrium nuclear composition can be established in the presence of an electron reservoir. This equilibrium is usually considered at high temperatures [2]. The problem of the equilibrium nuclear composition of compressed matter at low temperatures is of fundamental interest [3]. Low temperatures in this problem are temperatures below mc^2, where m is the electron mass. The relativistic electron gas at these temperatures is strongly degenerate. The value $\sim mc^2$ also corresponds to the characteristic scale of the difference between the energies per nucleon for various nuclei and isotopes.

Equilibrium at zero temperature corresponds to the minimum of the energy E of matter in the volume V at a given nucleon chemical potential μ. Let us first disregard the Coulomb energy. In this case, the potential that should be minimized, $\Omega = E - \mu N$, is given by the expression

$$\Omega = \sum_{ZA} M_{ZA}c^2 N_{ZA} + E_e(N_e/V)V - \mu N, \qquad (1)$$

where N_{ZA} and M_{ZA} are the number and mass of the ZA nuclei, E_e is the energy of the electron gas, $N_e = \sum_{ZA} ZN_{ZA}$ is the number of electrons, and $N = \sum_{ZA} AN_{ZA}$ is the number of nucleons. The neutrino contribution should be neglected (see [1]).

Let us vary potential (1) with respect to the number of isotopes

$$\delta\Omega = \sum_{ZA}(M_{ZA}c^2 + Z\mu_e - A\mu)\delta N_{ZA},$$

where μ_e is the electron chemical potential. A state is equilibrium if the condition $\mu = (M_{ZA}c^2 + Z\mu_e)/A$ is satisfied for all nuclei existing in a given phase. For the stability of the state, the inequality $M_{Z'A'}c^2 + Z'\mu_e - \mu A' > 0$ should be valid for all other nuclei. In this case, the addition of these nuclei, $\delta N_{Z'A'} > 0$, leads to an increase in Ω. It is clear that, when μ_e varies in a certain interval, only one type of isotope with the minimum value of μ corresponds to equilibrium.

The equilibrium nuclear composition at high temperatures includes a number of isotopes and their concentration determined by the chemical equilibrium conditions is a continuous function of temperature and pressure [2]. At a zero temperature, monoisotopic phases should be realized and a change in the composition $ZA \longrightarrow Z'A'$ occurs at the critical pressures corresponding to the electron chemical potentials $\mu_e = \mu_e^c$:

$$\mu_e^c = \frac{A'M_{ZA} - AM_{Z'A'}}{Z'A - ZA'}c^2. \qquad (2)$$

These transitions are accompanied by jumps in the nuclear component of the mass density

$$\Delta\rho = \frac{ZA' - Z'A}{AZ'}\rho,$$

and the electronic density remains unchanged.

An analysis of the parameters of 3180 known isotopes [4][1] shows that only 14 nuclei (see the table) can appear on the phase diagram of matter at a zero temperature. The possibility of existing short-lived isotopes that have not yet been discovered and could compete with isotopes presented in the table is not excluded. However, the current status of the theory cannot provide a certain reliable conclusion on this problem (^{118}Kr, ^{120}Sr?).

[1] Isotope masses were presented in [4] and the masses of the Z electrons should be subtracted from these quantities for the problem under consideration.

Table of isotopic phases

Isotope	μ_e^c	Δ	μ_e^*	a
$^{7}_{1}H$	74	^{40}Mn:9.2	16	20
$^{10}_{2}He$	72	^{7}H:0.9	32	24
$^{40}_{12}Mg$	61	^{51}Cl:2.3	45	55
$^{49}_{16}S$	55	^{51}Cl:0.4	46.7	67
$^{53}_{18}Ar$	54	^{49}S:0.6	47	71
$^{78}_{28}Ni$	26	^{79}Cu:1.5	47.4	170
$^{80}_{30}Zn$	21	^{78}Zn:1.4		220
$^{82}_{32}Ge$	15	^{80}Ge:0.2		300
$^{84}_{34}Se$	11	^{86}Kr:0.7		420
$^{66}_{28}Ni$	8.1	^{60}Fe:1.2		550
$^{64}_{28}Ni$	5.2	^{62}Ni:0.3		850
$^{58}_{26}Fe$	3.9	^{64}Ni:0.9		1100
$^{62}_{28}Ni$	1.9	^{58}Fe:0.4		2300
$^{56}_{26}Fe$	0	^{62}Ni:1.5		

Note: μ_e^c corresponds to the transition with an increase in the pressure, Δ is the minimum energy of a nuclear impurity, μ_e^* corresponds to the appearance of neutrons in the phase, a is the distance between the nearest nuclei in the bcc lattice at the minimum pressure. The energies and lengths are given in units of mc^2 and 10^{-13} cm, respectively.

The neutron Fermi gas appears in the system at $\mu > Mc^2$, where M is the neutron mass. This occurs in the ^{78}Ni phase at $\mu_e > \mu_e^* \approx 47.4mc^2$. A further increase in pressure leads to the filling of the neutron Fermi sphere.

The Coulomb corrections give rise primarily to the ordering of the nuclear subsystem of almost all phases in the form of a crystal lattice. When the spatial modulation of the electron density is disregarded, the situation is completely equivalent to the problem of the Wigner crystal. According to the known results [5], the nuclei are most probably located at the sites of a bcc lattice. The Coulomb energy of the structure consisting of the ZA nuclei differs from the energy of the lattice with localized electrons by the change $e \longrightarrow Ze$:

$$E_C = -\lambda_F Z^2 e^2 (4\pi N_{ZA}/3V)^{1/3} N_{ZA}, \qquad (3)$$

where $\lambda_F = 1.79186$ is the constant calculated by Fuchs [6]. The Coulomb contribution given by Eq. (3) provides small corrections to the nucleon chemical potential and pressure. It is easy to verify that the inclusion of these corrections leads to small, $\sim 10^{-3}$, relative jumps in the electron density in the transitions under consideration and to small, $\sim 10^{-3}$–10^{-2}, shifts of the critical pressure points.

The effects of the quantum delocalization of the nuclei become important in the maximum-pressure phases. Indeed, a comparison of the kinetic energy, $\sim\hbar^2/AM(\delta x)^2$, of the motion of the nuclei near the site of the lattice with the characteristic change in the energy of the Coulomb interaction between nuclei, $\sim z^2 e^2 (\delta x)^2/a^3$, gives

$$\beta = \frac{\langle(\delta x)^2\rangle}{a^2} \sim \frac{\hbar}{Ze\sqrt{AMa}} \sim \frac{1}{Z\sqrt{A}}\sqrt{\frac{ma_0}{Ma}}, \qquad (4)$$

where $a_0 \sim 10^{-8}$ cm is the atomic size. The estimate of the parameter β shows that the ^{7}H phase is likely a Fermi liquid already after the transition from the ^{10}He phase ($\beta \sim 1$). Quantum melting ($\beta \sim 0.3$) is possible in the ^{10}He phase. In other phases, $\beta \sim 10^{-3}$–10^{-2}.

Thermally activated impurities of other nuclei appear in equilibrium matter at low temperatures. The table presents the impurities with the minimum energy in each phase near the transition point with an increase in the pressure. With a further increase in temperature, as well as for the case of normal phase transitions, the lines of the isotopic transitions should end at critical points.

The phases discussed above must be realized at the final stage of the cooling of stars. At this stage, nuclear reactions inside the phases cease and the kinetics of the isotopic phase transitions is the single source of the energy release until the phase interfaces stop in the equilibrium positions.

I am grateful to A.Ya. Parshin for stimulating discussions.

REFERENCES

1. L. D. Landau and E. M. Lifshitz, *Course of Theoretical Physics*, Vol. 5: *Statistical Physics*, Part 1 (Nauka, Moscow, 1995; Pergamon, Oxford, 1980).
2. D. A. Kirzhnits, Usp. Fiz. Nauk **104**, 489 (1971) [Sov. Phys. Usp. **14**, 512 (1971)].
3. Ya. B. Zeldovich and I. D. Novikov, *Theory of Gravitation and Evolution of Stars* (Nauka, Moscow, 1971) [in Russian].
4. http://www.nndc.bnl.gov/amdc.
5. W. J. Carr, Phys. Rev. **122**, 1437 (1961).
6. K. Fuchs, Proc. Roy. Soc. (London) A **151**, 585 (1935).

Translated by R. Tyapaev