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On Wetting Transition in Superconductivity

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Abstract—The existence of two solutions for proximity-induced superconductivity described by the Ginzburg–Landau theory is established in the general case. A first-order phase transition can occur between these states, resulting in superconductive wetting.

It has been commonly believed that a critical wetting transition must occur when two superconductors characterized by different critical temperatures are brought into contact: as the critical magnetic field strength is approached, the proximity-induced superconducting layer gradually expands into the bulk of the weaker superconductor [1]. However, it was found that the normal state of aluminum brought into contact with tin or tantalum can be substantially overcooled [2].

To resolve the controversy, we have analyzed the proximity effect by applying the Ginzburg–Landau theory. We have found that a first-order phase transition can occur in the proximity-induced superconducting layer, with a jump in the wetting-layer thickness, and that wetting can take place only in the phase characterized by the larger thickness.

The possibility of diverse junction behavior can be demonstrated under quite general assumptions. Suppose that the wetting-layer thickness L increases indefinitely as the magnetic field H approaches the critical value H_c. Then, the system can be described by a macroscopic model.

The order parameter has the equilibrium bulk value almost everywhere in the thicker layer, deviating from it only within a distance on the order of the coherence length ξ from the layer boundaries. Owing to exponential decay of these deviations into the bulk of the layer, the NS interface interacts with the junction. Thus, the energy of the proximity-induced superconducting layer can be represented as (cf. [3])

$$\sigma_{NS} + \sigma_{SS} + \frac{H^2 - H_c^2}{8\pi}L + \beta e^{-L/\tilde{\xi}}, \tag{1}$$

where $\tilde{\xi} = \sqrt{2}\,\xi$ (in conventional notation [4]), σ_{NS} is the energy of the NS interface, and σ_{SS} is the junction energy. Note that, if the Ginzburg–Landau parameter κ is close to $1/\sqrt{2}$, then the term proportional to $\exp(-L/\delta)$ must also be included to allow for magnetic field penetration.

When the NS interface is repelled from the junction ($\beta > 0$), the following wetting law is obtained by minimizing the energy given by (1):

$$L \propto \xi \ln\frac{H_c}{H - H_c}. \tag{2}$$

When the NS interface is attracted to the junction ($\beta < 0$), the wetting-layer thickness remains finite as the field strength approaches the critical value. This state can be described only by microscopic theory.

Suppose that, in addition to the "long" state, there exists a locally stable proximity-induced superconductive state characterized by a finite thickness. A first-order phase transition between these states of the proximity-induced superconducting layer can occur across a curve $H_s(T) > H_c(T)$. Since both phases have the same symmetry, the curve $H_s(T)$ can terminate at some critical point, as is common for first-order transitions. However, when the curves $H_s(T)$ and $H_c(T)$ meet at some point (wetting point) (H_{WT}, T_{WT}), the system exhibits an uncommon behavior that can also be analyzed in the framework of a macroscopic model.

On the phase-transition curve, the energy of the "long" (wetting) solution given by (1) equals the energy σ of the "short" (nonwetting) one. In the vicinity of the wetting transition point, the linear magnetic-field dependence of σ can be neglected as compared to the stronger dependence expressed by (1). The temperature dependence can also be neglected by setting $T = T_{WT}$ everywhere except for

$$\sigma - \sigma_{NS} - \sigma_{SS} = C(T - T_{WT}), \tag{3}$$

where the constant $C > 0$, in agreement with the phase diagram obtained by computing the Ginzburg–Landau equation (see below). Minimizing (1) with respect to L

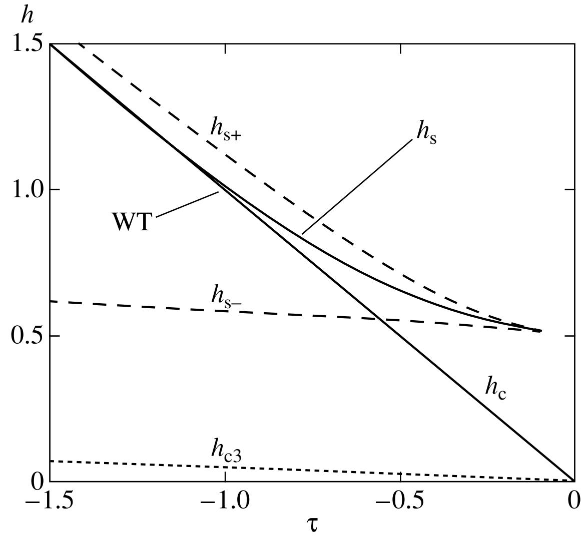

Fig. 1.

and matching the energy of both states, we obtain the following expression describing the behavior of the phase equilibrium curve in the neighborhood of T_{WT}:

$$H - H_c \propto -\frac{T - T_{WT}}{\ln(T - T_{WT})}. \tag{4}$$

Now, we analyze the proximity effect and explore the applicability of the Ginzburg–Landau theory to the phase transition. We consider the simplest case of states that are uniform in the junction plane. Then, the Ginzburg–Landau equations reduce to two equations for the ψ-function and the vector potential A (Eqs. (46.8) and (46.9) in [4]).

To formulate boundary conditions at the junction, we must take into account only the term linear in the order parameter ψ of the weaker superconductor,

$$\lambda(\psi^* e^{i\alpha} + \psi e^{-i\alpha}). \tag{5}$$

Here, λ represents the coupling between the superconductors, and α is the value of the order-parameter phase for the stronger superconductor near the junction.

When $\lambda < 0$, the order-parameter phases are equal at the boundary; when $\lambda > 0$, they differ by π (this case is known as π-junction [5]). We note here that the Ginzburg–Landau theory does not make any qualitative distinction between these states. The only observable distinction is the presence of a ubiquitous vortex carrying one-half of the magnetic-flux quantum near the curve where λ changes sign in inhomogeneous structures.

By virtue of gauge invariance at field strengths that are low for the stronger superconductor, we can set $\alpha = 0$. Furthermore, near the bulk phase transition point, where all characteristic lengths in the weaker superconductor increase, the finiteness of the magnetic penetration depth can be ignored. Then, the boundary condition naturally derived by varying the energy reduces to

$$\frac{\hbar^2}{4m}\partial_x\psi = \lambda. \tag{6}$$

The x axis is directed along the normal to the boundary into the weaker superconductor. Note that the ψ-function and its derivative at the boundary have opposite signs. In dimensionless quantities (see [4]), the boundary condition is rewritten as

$$\psi' = \kappa\Lambda, \quad \Lambda = 2\lambda\frac{\sqrt{mb}}{\hbar|a|}. \tag{7}$$

The structure of the boundary between the superconductors is readily calculated in the absence of magnetic field:

$$\psi = -\coth\frac{\kappa(x+c)}{\sqrt{2}} \tag{8}$$

(it is assumed that $\lambda > 0$). The constant c is determined by using the boundary condition. The corresponding SS-junction energy contained in (1) is

$$\sigma_{SS} = \frac{H_c^2\xi}{3\sqrt{2}\pi}(1 - |\psi_0|^3), \tag{9}$$

where ψ_0 is the value of the ψ-function at the boundary.

We have performed a numerical analysis of the proximity effect and determined the domain of stability for the solutions obtained. Depending on κ, magnetic field, and temperature, there exist either one or two (locally) stable solutions, and a phase transition between them can occur.

Figure 1 shows the phase diagram for proximity-induced superconductivity in aluminum calculated for $\kappa = 0.02$ in the case when the wetting transition occurs in the neighborhood of the critical point T_c. The phase diagram is universal in the coordinates $h = H/H_{WT}$ and $\tau = (T - T_c)/(T_c - T_{WT})$. The equilibrium solution below the transition curve $h_s(\tau)$ corresponds to wetting.

Figure 2 shows the effective thickness of the surface solution

$$L = \int_0^\infty |\psi|^2 dx$$

plotted versus magnetic field strength at a temperature above the wetting transition point T_{WT} $\tau = -0.38$.

At field strengths above the curve $h_s(\tau)$, the equilibrium solution remains its finite thickness. When the stability boundary for the "short" solution h_{s-} lies below h_c, superconductivity can penetrate into the bulk of the weaker superconductor only after a critical nucleus forms either in the bulk or at the junction under the so-called incomplete wetting conditions.

When the Ginzburg–Landau parameter is small, both short and long solutions obtained in the neighborhood of the transition point admit analytical description. In particular, the coordinates of the point (H_{WT}, T_{WT}) can be found. The short solution is characterized by the thickness $\sqrt{\xi\delta}$, and the corresponding ψ-function amplitude is small ($\propto\sqrt{\kappa}$, see the considerations that follow after Eq. (46.14) in [4]). The energy of the short solution is small as compared to that of the SS-junction energy σ_{SS} given by (9) and the NS-interface energy given by (9) with $\psi_0 = 0$ (see [4, Eq. (46.14)]). Therefore, the wetting transition occurs at the point defined by the condition $\sigma_{SS} + \sigma_{NS} = 0$, which yields the value of the ψ-function at the SS junction for the long solution at the wetting transition point, $|\psi_0| = 2^{1/3}$, and $\Lambda = 2^{1/6} - 2^{-1/2}$.

The parameter Λ increases as $(T_c - T)^{-1}$ toward the critical temperature, and wetting must always be observed in the neighborhood of the superconducting transition, in agreement with [1]. The wetting transition can occur in the close neighborhood of the critical temperature, where it can be described by the Ginzburg–Landau theory, only if the parameter λ is small, i.e., when the superconductors are weakly coupled. If λ is too small, (the domain of equilibrium wetting lies too close to T_c and $\xi(T)$ is on the order of ξ_0^2/a, where a is an atomic distance), then the term linear in the ψ-function must be retained in boundary condition (6) (see [6]). In this case, the phase diagram may not contain the domain of wetting solution.

A numerical analysis shows that the wetting transition discussed here can occur only in type I superconductors, whereas there exists only one surface solution for type II superconductors, which has a finite thickness until H_{c2} is reached.

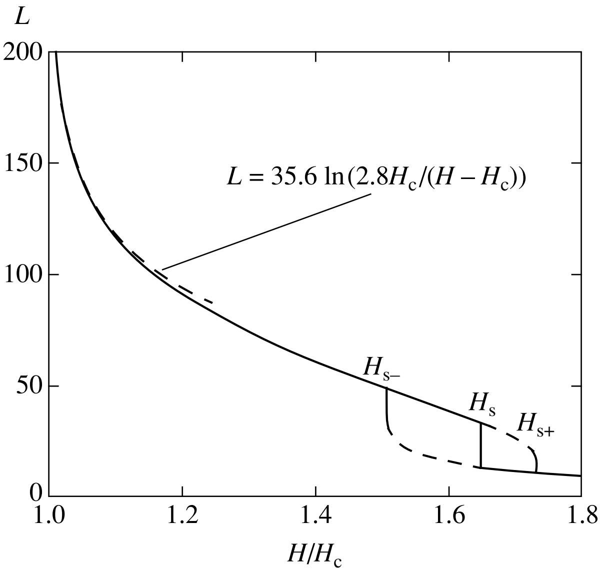

Fig. 2.

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