The Landau-Lifshitz domain structure in a magnetic field

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The variation of the period of the Landau-Lifshitz ferromagnetic domain structure is found in an external magnetic field directed at an arbitrary angle to the anisotropy axis.

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This paper considers the behavior of the domain structure of uniaxial ferromagnets with low anisotropy—the Landau-Lifshitz^[1] structure—in an external magnetic field directed at some angle to the anisotropy axis. The corresponding problem for the Kittel structure has been solved only for the case of a field directed along the surface of the specimen. In the presence of a perpendicular component, as was shown by Provorotskii, ^[2] the boundaries between the domains in the Kittel structure must curve near the surface, and therefore the solution involves great complications. In our case the nature of the structure does not change, and the problem can be solved exactly for an arbitrary direction of the field.

It is clear that in a state of thermodynamic equilibrium the interdomain boundaries are aligned along the field and the magnetic moment of a boundary is directed along the component of the external magnetic field parallel to the surface, since the surface tension is smallest in this case. [3] The opposite direction of a boundary moment corresponds to a metastable state. and therefore, as is usual in such situations, nuclei with the right direction of moment must form in the boundary. Sufficiently large nuclei lower the energy; therefore they grow without hindrance, and the boundary transforms to a state with the minimal energy. The energy of formation of a critical nucleus is considerably smaller than the Curie temperature, since it is due to magnetic energy and anisotropy energy. Therefore at ordinary temperatures the boundary should transform easily. We shall not investigate the kinetics of this transformation but shall restrict ourselves to a determination of the period of the structure under conditions of complete thermodynamic equilib-

In a magnetic field the Landau-Lifshitz structure must change, as is depicted in Fig. 1. We introduce a Cartesian system of coordinates (x,y,z) whose z axis is directed along the anisotropy axis, which is assumed to be perpendicular to the surface of the plate. We direct the x and y axes parallel and perpendicular, respectively, to the interdomain boundaries. The external magnetic field will have two components: the projections on the z axis and on the x axis, x and x and x respectively. We shall describe the direction of the magnetic moments in the closure domains by angles x and x of a spherical coordinate system whose polar axis coincides with the x axis and whose angle x is measured from the x axis.

The values of the angles θ , ψ_1 , and ψ_2 (see Fig. 1) are determined by the condition of continuity of magnetic flux:

$$\begin{split} \cos\theta = & II_{\parallel}/4\pi M = \eta, \\ \text{ctg } \psi_{i} = & \frac{\cos\theta_{o} + \cos\theta}{\sin\theta\cos\phi}, \quad \text{ctg } \psi_{2} = \frac{\cos\theta_{o} - \cos\theta}{\sin\theta\cos\phi} \end{split}$$

here M is the saturation moment, and θ_0 is the angle of deviation of the moment in a main domain from the z axis. It is obvious that

$$\sin \theta_0 = H_{\parallel}/\beta M = h$$

where β is the anisotropy constant.

We shall find the angle φ from the condition of minimum energy of a closure domain at constant period. By using Maxwell's equations and also neglecting terms of order β^2 , it is not difficult to transform the expression for the energy density [4]

$$^{1}/_{2}\beta \left(M_{x}^{2} + M_{y}^{2} \right) - HM - H^{2}/8\pi$$

(H is the magnetic field, $\mathbf{M} = (M_x, M_y, M_z)$ the magnetic moment) to the following form:

$$^{1}/_{2}\beta (M_{x}^{2}+M_{y}^{2})-H_{ij}M_{o}.$$

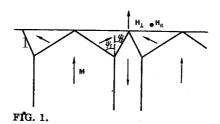
Here $H_{ii} = (H_{ii}, 0, 0)$, and M_0 is the magnetic moment density in the structure depicted in Fig. 1.

Now, taking into account that the concentration of domains with magnetic-moment direction "up" is $(1+\eta)/2$, one easily finds the energy connected with formation of a closure domain:

$$[(1-h^2)^{\frac{n}{2}}(1+\eta^2)-2\eta^2][1-\eta^2+h^2-2h(1-\eta^2)^{\frac{n}{2}}\sin\phi](1-\eta^2)^{-\frac{n}{2}}(\cos\phi)^{-1}.$$

Here and below, we write only factors that depend on the field. The minimum of this expression with respect to the angle φ occurs when

$$\sin \varphi = 2h(1-\eta^2)^{\frac{n}{2}}/(1-\eta^2+h^2)$$
.



The exit energy is then

$$[(1-h^2)^{-h}(1+\eta^2)-2\eta^2](1-\eta^2-h^2)(1-\eta^3)^{-h}$$

Finally, knowing the surface tension of a domain boundary, [5] we can write the expression for the period a:

$$a = \left\{ \frac{(1-\eta^2)^{\frac{t_1}{h}} [\, (1-h^2)^{\frac{t_1}{h}} - (\pi/2 - \theta_0) \, h \,]}{[\, (1-h^2)^{\frac{t_1}{h}} (1+\eta^2) - 2\eta^2 \,] \, [\, 1-\eta^2 - h^2 \,]} \right\}^{\frac{t_1}{h}}.$$

For $H_n = 0$, the variation of the period with the field was obtained earlier.^[5] For $H_1 = 0$,

$$a=[(1-h^2)^{-h}-(\pi/2-\theta_a)h]^{-h}(1-h^2)^{-h},$$

this differs with respect to the exponent of the second expression in parentheses from the case $\beta \gg 4\pi$, where the exponent is $-\frac{1}{2}$. In particular, for h-1 (i.e. $H_0 + \beta M$) the period approaches the finite value $3^{-1/2}$.

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